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Student voices: implications for teaching mathematics

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Abstract: We report on an international study of students' ideas about mathematics and how they view its relevance to their further studies and their careers. About 1200 students enrolled in mathematics subjects at five universities in different countries responded to three open-ended questions. The responses were coded using phenomenographic methods and grouped into a hierarchy of conceptions. We used a qualitative methodology in preference to a closed form survey in order to draw out the students' ideas rather than imposing those of the researchers. We present the categorisations found and implications for learning and teaching.

1 Introduction

As practising mathematicians, we are passionate about what we do and would like to see our students enjoying mathematics as well. We see it is an important area of study and work, as a major in itself and as an integral part of other disciplines such as engineering. But what of our students? Do they also believe that mathematics is an important part of their studies? Do they view studying mathematics as simply a compulsory requirement, or is it incorporated in their wider views of life and work? This paper examines our research on these questions and makes recommendations for the design of university mathematics teaching and learning.

We are carrying out an extended international study around these ideas and we report here on the qualitative aspects of our project, based on the voices of the university students themselves. The current stage builds on phase 1 – reported in Reid *et al.* [1] – where we interviewed 22 mathematics undergraduate students in depth and found that they had three qualitatively different conceptions of mathematics. We have broadened the study to include a wider group of participants as well as to include questions that investigate student ideas of the relevance of mathematics in their future studies and career. We now have 1,200 participants from 5 universities, indeed 5 continents. The participating universities also represent diverse cohorts, for instance, the University of Technology, Sydney, attracts a high number of international students from South East Asia. Our data gives us the capability to investigate cross-cultural differences in a later phase.

The written survey consisted of general demographic questions and three open-ended questions, which were:

What is mathematics?

What part do you think mathematics will play in your future studies?

What part do you think mathematics will play in your future career?

This paper reports on the findings of the qualitative phase of the project.

1.1 Background and methodology

Our interest in teaching mathematics has led us to wonder about the views of our students. Literature searches and discussions with other university mathematics educators revealed that there has been little investigation of this, even though mathematics provides foundation skills for a variety of disciplines, from engineering, architecture or teaching to mathematics as a major. We found some examples of research, such as Crawford *et al.* [2] who undertook a study of first year students at an Australian university, and Leitze [3] who assessed the views of mathematics major students. Nonetheless, there has been limited interest in the voices of the students themselves; samples have been relatively homogeneous; and work on investigating higher-level mathematics learning is rare. There is also a gap in the literature investigating students' transition to the workplace and Wood [4] has shown that there is little research into graduates. This study addresses these concerns from the viewpoint of students.

Participants in our project came from five universities and we targeted first and final year students in order to explore the differences between stages of study. They were all studying mathematics subjects and were predominantly drawn from degree programs with a reliance on the mathematical sciences. Students were invited to participate in the project, asked to provide some demographic data (age, sex, language background) and information about their program (year, degree, major). They were given about three lines in which to write their responses for each of the questions listed above.

To analyse their responses we used the approach of phenomenography, which looks at how people experience, understand and ascribe meaning to a specific situation or phenomenon [5]. It is a qualitative methodology that is often used to describe the experience of learning and/or teaching. The outcome of a phenomenographic study is a hierarchical set of logically related categories, from the narrowest and most limited to the broadest and most inclusive. Phenomenography defines aspects that are critically *different* within a group involved in the same situation.

Initially, the responses from around 400 of the students were randomised so that we could not tell from which university or group they came. Four team members then read all the responses and individually suggested ways to code them. We were guided but not restricted by the results reported in [1]. We then met and refined our categorisations. We individually coded all the responses, met again, discussed differences and resolved them. Those student responses where we could not agree (although few) were omitted from analysis. Not all responses could be coded into our hierarchy of conceptions, for example, those student responses that stated that the use of mathematics in the student's future studies or career was "a little" or "a lot". After the first 400 were agreed upon, the remainder of the responses were coded according to our schema. Although quantitative analysis of the data is currently being undertaken, we felt that it was interesting to present our qualitative results indicating the range of conceptions.

2 Results

We have presented the results in the form of quotes from students and analysis of the quotes. The quotes are as written by the students, although spelling and minor punctuation has been corrected for ease of reading. We have chosen quotes from the same university (300 participants) to illustrate the fact that students within the same group will hold quite different ideas about mathematics and how they will use it. However, quotes from other universities show the same general range.

2.1 What is mathematics?

Consider these responses to the question: *What is mathematics?*

1. It is an infinite universe of possibilities that is presented to the student at first as a tiny speck of dust – that grows and grows into something large enough to provide answers to virtually anything of a natural (i.e. not moral) nature.
2. An abstract concept defined by a small number of laws but regulated by the laws of pure logic used to model the real world.
3. In real world, lots of problems can be defined by mathematics model. In other words, math can solve many problems in real life. Math is the basic knowledge for all the scientific fields (physics, chemistry, economics, psychology, computer science and so on). Math is a kind of language, which is abstract and beautiful.
4. As far as I am concerned it is the study of tools that will allow to best approximate real life phenomena.
5. All the ways of calculating problems and all kind of measurements.
6. It's numbers and calculating.
7. It's a pain in the ass, but is very useful when you have it figured out.

Notice that the quotes demonstrate the different ways that students are experiencing mathematics in our classes. We have categorised these as:

Life: Quote 1. In this conception, students view mathematics as an integral part of life and as a way of thinking. They believe that reality can be represented in mathematical terms but in a more comprehensive way than the modelling conception (below). Their way of thinking about reality is mediated by mathematics. They may make a strong personal connection between mathematics and their own lives. This is the highest level of conception identified.

Modelling/Abstract: Quotes 2 and 3. Both these quotes show abstract and modelling ideas. The students experience the structure of mathematics in quite a different way to quotes 5 and 6. The modelling conception links mathematics to the physical world. Indeed the people who hold this conception make strong connections between mathematics and the physical world. This world can be described, perhaps imperfectly, by mathematics. The abstract conception shows mathematics as a logical system or structure, perhaps even a kind of game of the mind. We have placed Quote 4 into the modelling category, but it is a borderline case between modelling and components.

Components (Toolbox): Quote 5. Here mathematics is seen as a toolbox to be dipped into when necessary to solve a problem. It may also be viewed as a collection of isolated techniques unrelated to real world applications. Students mention formulas, equations and laws. There is no reference to structure.

Number: Quote 6. In this conception, students consider mathematics to be connected with numbers and calculations. Mathematics is manipulation with numbers with no essential advance beyond elementary arithmetic. Students mention numbers, calculations, sums and basic operations. This is the lowest level identified.

Quote 7 may be amusing – but it does not fit into the above categorisation.

The hierarchy of conceptions is such that the higher levels subsume the lower levels. Students who talk about higher-level thinking may also describe components of mathematics, but not vice versa.

2.2 What part do you think mathematics will play in your future studies?

Again, we begin with quotes from our students:

1. Because I study math you understand that math is the most important part on my studies!
2. Mathematics is the most important science of foundation. It's the base of any other sciences. In future, mathematics will play important aspect for my future.
3. It will develop logical thinking in abstract situations. It will help me solve most engineering problems.
4. Major part! It will probably be the tool that I will use the most in my future.

Life: Quote 1. Here mathematics is seen as playing an overwhelming role in future studies. It may be because the students are mathematics majors or contemplating a higher degree in mathematics, or simply that students believe it to be such an integral part of their studies that it really cannot be separated from them.

Foundation/Analytic: Quote 2 – the foundation conception considers mathematics as a foundation for further study in mathematics or in other studies. The purpose of studying mathematics is to provide underlying principles in an area or discipline. Quote 3 – the analytic category includes the idea that studying mathematics develops logical thinking and problem solving skills. The problem solving skills here are generic rather than particular. Mathematics is useful for deepening understanding in various areas or disciplines.

Components (Toolbox): Quote 4. Students who hold this conception have the idea that mathematics will be used as a toolbox to be reached into when needed in their studies. It describes a point of view about the role of mathematics, rather than the depth or complexity of the mathematics itself. Tools may be simple or very intricate but they remain as tools. In this view, certain isolated skills or techniques are regarded as relevant and used as needed.

2.3 What part do you think mathematics will play in your future career?

Here are some further quotes from students:

1. Mathematics shows how to think logically and abstractly and no matter what I end up doing with my life I will be better at it because I have gone far in mathematics.
2. Mathematics always is an essential part of one's career irrespective of the job that one might have. It is a building block which lays the sound foundations and continual reinforcement to one's career.
3. I cannot answer that question as of right now since I don't know what I will be doing in the future just yet. But in almost every career mathematics has a certain amount of importance.
4. Analysis of data, creation of algorithms.
5. Probably nothing. Maybe I'll only use basic calculation. \int does not help.
6. It will play a huge role in my future career because I am focusing on an actuarial career.
7. In network engineering of fibre optics? I already know enough math for what I want to do. I know from my previous job at a Telecom company.
8. But, I am convinced that using softwares (such as Excel) to do numerical calculations will occur more often. Courses in numerical analysis should therefore be more present in the curriculum.
9. I really don't have any ideas. Sorry!

Many students were unclear about this question and many answered as in quote 9. Very few answered as specifically as quote 7; this student clearly had been working in industry and felt that s/he knew enough mathematics already. Quote 6 is common but students were unable to articulate why or how the mathematics would be used in their career. Several students expressed the views from quote 8 and this has important implications for teaching. In many of the quotes above students are unsure and this shows in their word use, such as *Maybe* (quote 5) and *certain amount* (quote 3). These are the conceptions that we identified:

Life: Quote 1. Here mathematics plays an overwhelming role in their future career and/or life. It has changed their way of thinking.

Foundation/Generic: Quotes 2 and 3. This conception considers mathematics as a foundation for further study in mathematics or in other studies. The purpose of studying mathematics is to provide underlying principles in another area or discipline. Here the perception is that mathematics is used in all careers and that it is useful for deepening understanding in various areas or disciplines.

Components/Toolbox: Quote 4. Students who hold this conception have the idea that mathematics will be used as a toolbox to be used when needed in their careers. It describes a point of view of the role of mathematics, rather than describing the mathematics itself. In this view, certain isolated skills or techniques are regarded as relevant and used as needed.

3 Implications for teaching and learning

These results give us an idea of the conceptions of students in our classes. The challenge for us is to acknowledge that they hold greatly differing ideas – even though some may seem overly simplistic – about what mathematics is and how they believe they will use it in their future study and careers. While the conceptions quoted here are from one university, they illustrate the views from the larger group.

One result exposed in this study is a widespread uncertainty about the role of mathematics, particularly within future careers. Preliminary statistical analysis showed that in relation to their studies, around 40% of the participants viewed mathematics simply as a toolbox or had an unformed notion that it was somehow necessary for their career. Similar results were found regarding how they view its specific role in their career, around 50% in this case. Ideally, students should have an understanding of how mathematics may be used, especially for career-directed students such as engineers. Career education should therefore be integrated more closely into their studies, rather than being an add-on in the final year, so we can help students to focus their learning throughout their degree programs. This is supported by a study by Kent and Noss [6], which shows that professional engineers need higher conceptions of mathematics and certainly a clearer idea of how the mathematics will be used.

The revealing of the categories of conceptions – and their hierarchical nature – suggests several learning and teaching strategies. By the time they enter university, students have developed conceptions of mathematics that may be further reinforced or changed during tertiary study. Our belief, naturally, is that we would like the majority of our students to hold higher conceptions by the time they finish an undergraduate degree, and we would aim to achieve this through a curriculum that allows them to develop those understandings. One of the most powerful learning techniques is to show students that these categories exist! It can be useful to have class discussions about the fact that different people hold different conceptions of mathematics – students, lectures, practicing mathematicians – and that people have valid reasons for holding those conceptions.

So how else can we provide students with greater understanding? One way is to educate *ourselves* as to how it will be used, for example, through contact with industry and professional associations. Then our teaching materials and assessment practices need to reflect this knowledge by emphasising the development of higher-level skills, through the use of real materials, situations and computing tools. We can design learning tasks that model the way mathematicians work in industry and academia in order to give students a better idea of the way mathematics could be used in a career. As our concepts of student views have developed alongside the research, we have designed teaching and assessment materials along these lines (for example [7], [8] and [9]).

Modelling of professional work situations, such as fieldwork, working in teams, writing reports and so on, is useful for the development of students' perceptions of work. To quote two students who had watched one of our statistics videos [7], "*For the first time in my life I saw and understood what the jobs that I might do when I graduate actually look like*", and, "*That's the first time it's convinced me that we actually use that [probability] in real life*". An essential task for the lecturer is to make such learning situations explicit, so that students can make the connections between the mathematics and their future professional careers.

There is evidence to show that students have varying ideas about mathematics and that these conceptions influence how they learn [10]. Many see mathematics as a collection of 'bits and pieces' that are useful, but are unable to identify (explicitly) abstract or modelling features, let alone any relevance to the world beyond university. Learning and teaching strategies, monitoring, and more defined career education can help address these issues.

4 References

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